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FLOW ABOUT A POROUS-SURFACED ROTATING DISK

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INTRODUCTION

AN ANALYSIS of the flow about a porous-surfaced rotating disk is performed in which account is taken of velocity slip at the porous bounding surface. The flow field within the porous material is also analyzed. Results showing the effect of velocity slip on the torque due to surface shear and on the velocity field are presented.

Although the no-slip boundary condition is widely utilized for fluid flows bounded by porous walls, recent experiments [1, 2] have verified the existence of a slip velocity. The experiments were performed in a parallel-plate channel with water and oil as the working fluids. The magnitude of the slip velocity was as high as 60 per cent of the mean velocity in some cases [2]. A macroscopic model for the slip velocity led to analytical predictions which were in excellent agreement with the experimental results [2], thereby lending support to the model. The same slip model will be used here in the analysis of the rotating disk.

A schematic diagram of the situation under study is pictured in the inset of Fig. 1. The rotating disk consists of a porous material backed by a solid wall. The disk is situated in an otherwise quiescent fluid environment. The porous material has thickness t and is assumed to be homogeneous and isotropic.

ANALYSIS

Consideration is first given to the porous material with a view to exploring the possible existence of a flow normal to the surface $z = 0$ (i.e. possible injection or suction). The flow field in the bulk of a porous medium is governed by Darcy's law provided that inertial effects within the medium are negligible, as will be shown to be the case in the present problem. The representation of Darcy's law appropriate to the porous rotating disk is

$$V_r = -(k/\mu)(\partial p/\partial r), \qquad V_\phi - r\omega = -(k/r\mu)(\partial p/\partial \phi),$$
$$V_z = -(k/\mu)(\partial p/\partial z) \qquad (1)$$

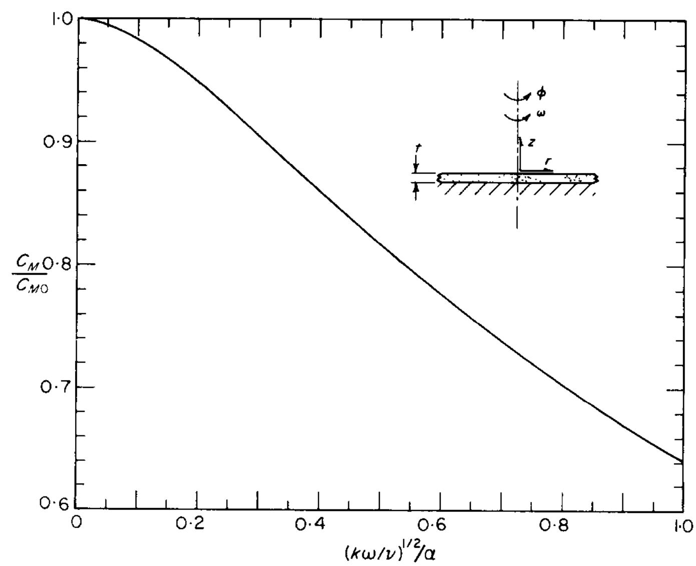

FIG. 1. Effect of velocity slip on torque.

where V_r, V_ϕ and V_z are with respect to fixed coordinates, and k is the permeability. In addition, the continuity equation is

$$\frac{\partial(rV_r)}{\partial r} + \frac{\partial V_\phi}{\partial \phi} + \frac{\partial(rV_z)}{\partial z} = 0. \qquad (2)$$

It may be observed that an equivalent statement of equations (1) and (2) is $\nabla^2 p = 0$.

The tangential velocity V_ϕ is equal to $r\omega$ since p is independent of ϕ (as are all other dependent variables). It is well established that the flow external to the disk obeys a similarity solution, and it is natural also to seek a similarity solution for the flow within the porous material. Consistent with the first and third members of (1), the similarity solution has the form

$$V_r = f(r), \qquad V_z = h(z) \qquad (3)$$

and to satisfy (2)

$$rV_r = c_1 r^2/2 + c_2, \qquad V_z = -c_1 z + c_3. \qquad (4)$$

The integration constants appearing in equation (4) may be determined by employing the conditions: (a) $V_z = 0$ at $z = -t$; (b) p (and, consequently, $\partial p/\partial r$) is continuous at $z = 0$. The first of these conditions yields $c_3 = -c_1 t$. In connection with the second condition, it may be noted that $\partial p/\partial r = 0$ in the flow external to the disk. Therefore, from the first members of (1) and (4), it follows that $c_1 r^2/2 + c_2 = 0$, so that $c_1 = c_2 = 0$ and, from above, $c_3 = 0$. Then, from the second of (4), it is seen that $V_z = 0$ within the bulk of the porous material.

Next, attention may be turned to the flow field external to the rotating disk. The starting point of the analysis is the complete Navier–Stokes equations and the continuity equation for steady, three-dimensional incompressible flow. A similarity solution for the velocity field may be sought in the form [3]

$$V_r = r\omega F(\eta); \qquad V_\phi = r\omega G(\eta);$$
$$V_z = (\omega\nu)^{\frac{1}{2}} H(\eta); \qquad \eta = z(\omega/\nu)^{\frac{1}{2}} \qquad (5)$$

the substitution of which into the aforementioned conservation equations yields

$$H''' = HH'' - (H')^2/2 + 2G^2, \qquad G'' = HG' - H'G \qquad (6)$$

with the auxiliary relation

$$H' = -2F. \qquad (7)$$

The boundary conditions will now be discussed. At the disk surface, the existence of a slip velocity is connected with the presence of a thin layer of moving fluid just beneath the surface of the porous material (in effect, a thin boundary layer). The fluid in this layer is pulled along (or retarded) by the flow external to the porous medium. The magnitude of the slip velocity depends on the properties of the porous material as well as on the magnitude of the velocity gradient which acts on the porous material. To characterize the slip velocities, the model that was previously employed in [1] and [2] is used

$$\frac{\partial V_r}{\partial z} = \frac{\alpha}{\sqrt{k}} V_r, \qquad \frac{\partial V_\phi}{\partial z} = \frac{\alpha}{\sqrt{k}}(V_\phi - r\omega) \qquad (8)$$

in which α is a dimensionless constant which depends on the porous material. At present, α is determined by experiment, as is the permeability k. For example, for the porous material of [2], $k = 5{\cdot}1 \times 10^{-7}\,\mathrm{cm}^2$ and $\alpha = 0{\cdot}1$.

In view of the already demonstrated result that $V_z = 0$ within the bulk of the porous material and of the thinness of the aforementioned slip boundary layer, it is reasonable to take $V_z = 0$ at the disk surface.* In the external flow, far from the disk surface, $V_r \to 0$ and $V_z \to 0$.

When the boundary conditions are recast into dimensionless form with the aid of equations (5) and (7), there follows

$$\eta = 0: \quad H = 0, \qquad H' = \frac{(k\omega/\nu)^{\frac{1}{2}}}{\alpha} H'',$$
$$G = 1 + \frac{(k\omega/\nu)^{\frac{1}{2}}}{\alpha} G' \qquad (9)$$
$$\eta \to \infty: \quad H' \to 0, \qquad G \to 0. \qquad (10)$$

The quantity $(k\omega/\nu)^{\frac{1}{2}}/\alpha$ is a slip grouping. When this grouping is zero, the boundary conditions reduce to those for the no-slip case.

The mathematical system consisting of equations (6), (9) and (10), is a two-point boundary value problem with a prescribable parameter $(k\omega/\nu)^{\frac{1}{2}}/\alpha$. Solutions were carried out by a forward integration technique on a CDC 6600 digital computer.

RESULTS AND DISCUSSION

The quantity which is, perhaps, of most direct technological interest is the shaft torque required to maintain steady rotation of the disk. This quantity is most conveniently determined by making use of a control volume having a surface which coincides with the plane $z = 0$. This choice permits the evaluation of the torque from a knowledge of the flow field external to the rotating disk, no consideration of the details of the flow within the disk being required. The tangential shear $\tau_{z\phi}$ acting on the plane $z = 0$ is expressible as $\rho r(\nu\omega^3)^{\frac{1}{2}} G'(0)$, and, with this, the torque M associated with the region $0 \leqslant r \leqslant r_0$ is

$$M = -\int_0^{r_0} r\tau_{z\phi} 2\pi r\,\mathrm{d}r = \tfrac{1}{2}\pi\rho r_0^4(\nu\omega^3)^{\frac{1}{2}}[-G'(0)] \qquad (11)$$

* V_z is not identically zero at $z = 0$ since $V_r \neq 0$ at $z = 0$.

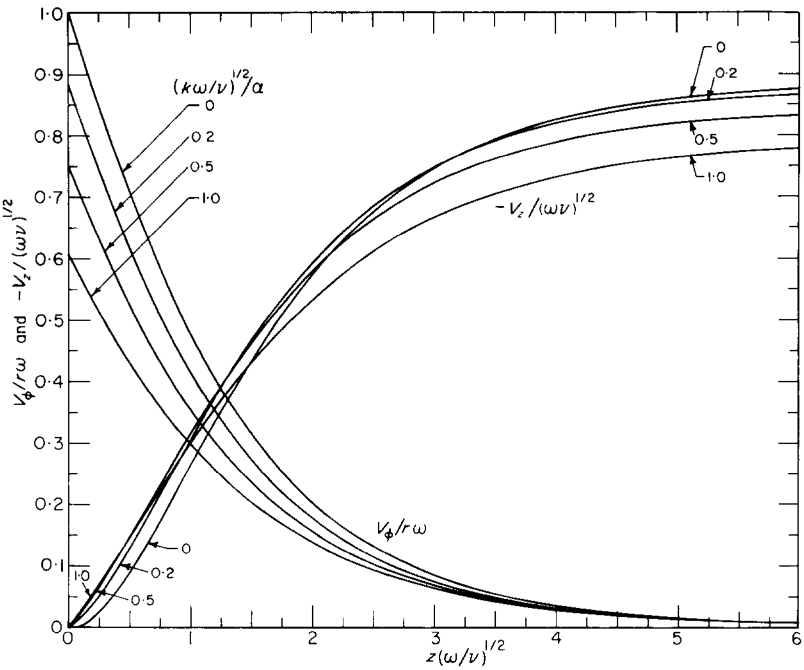

FIG. 2. Representative tangential and axial velocity profiles.

or, in terms of a dimensionless torque coefficient C_M,

$$C_M = M/\rho r_0^4(\nu\omega^3)^{\frac{1}{2}} = (\pi/2)[-G'(0)]. \tag{12}$$

The effect of velocity slip on the torque coefficient is shown in Fig. 1. The ordinate variable compares the torque in the presence of velocity slip to the torque in the absence of slip (the subscript 0 denotes zero slip). The abscissa is the slip grouping $(k\omega/\nu)^{\frac{1}{2}}/\alpha$. The figure indicates that substantial reductions in torque may occur as a result of slip. This finding suggests that for rotating components in general, it might be advantageous to purposefully design with the objective of accentuating velocity slip.

Further insights into the effects of slip are afforded by examination of representative velocity profiles. Figure 2 contains information on the tangential and axial velocity components (V_ϕ and V_z, respectively) plotted as a function of the axial coordinate z. In general, V_ϕ decreases monotonically with increasing distance from the disk surface, while $-V_z$ increases in the direction normal to the surface. The velocity slip is evidenced by the fact that $V_\phi < r\omega$ at $z = 0$. The extent of the tangential velocity slip increases with increasing values of the slip grouping. Since the tangential shear at the wall is proportional to the corresponding velocity gradient, the decrease in shear associated with velocity slip is readily apparent. The axial velocity field is not markedly affected by the slip in V_ϕ and V_r.

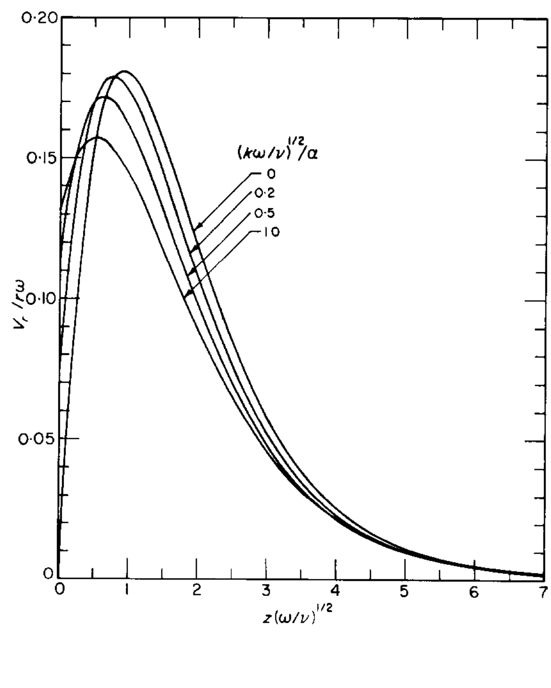

FIG. 3. Representative radial velocity profiles.

Representative radial velocity profiles are presented in Fig. 3. In general V_r is seen to increase with increasing z in the neighbourhood of the wall, attain a maximum, and then to decay asymptotically to zero at large z. The effect of slip is to diminish somewhat the magnitude of the velocity maximum and to shift its location closer to the wall.

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A DOUBLE-RAY TECHNIQUE FOR THE INVESTIGATION OF LIQUID BOUNDARY LAYERS

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1. INTRODUCTION

For the experimental verification of mass transfer theories, the measurement of the local and time-dependent concentration distribution within the boundary layer appears to be essential. Probe techniques, however, are for this purpose limited to measurements in the gaseous phase. For the very thin liquid boundary layers, only optical methods seem applicable. Indeed, interference methods have been adapted by Lin *et al.* [1] to a solid–liquid interface and by Jepsen *et al.* [2] to a gas–liquid interface. A great disadvantage of the interference method is that the direction of observation must be parallel to the boundary surface, and therefore only an averaged value along this direction is obtainable. Furthermore the boundary surface must be plane.

2. THEORETICAL BASIS AND DESCRIPTION OF THE METHOD

The use of a pH-indicator offers another possibility of measuring the concentration distribution. This method has been adapted to the case of mass transfer from the gas phase to a falling film [3, 4]. Here the direction of observation is perpendicular to the interface and the resolution in directions along and perpendicular to the interface is very high. The time average of the local concentration distribution was evaluated and from this the spatial distribution of the effective diffusivity. This gives a detailed picture of turbulent mass transfer.

Mass transport is obviously interrelated with the wave formation in the falling film. This calls for an extension of the pH-indicator method which would allow to recognize the correlation between wave dynamics and transport mechanism. A solution to this problem was found in a double-ray technique which was originally developed for the measurement of concentration profiles near a solid wall. The method was adapted to the falling film by T. Melin [5].

The new method makes use of the fact that many colour indicators—as well as a few fluorescent indicators—exhibit one or more so-called isosbestic (isoemissive) points (Fig. 1). When gaseous ammonia is absorbed in a falling film of a diluted strong acid, containing a minute amount (about 10^{-5} moles/l) of the fluorescent indicator acridine, two distinct regions are formed (Fig. 2): at the surface an